An exceptionally beautiful way to communicate over a classical channel

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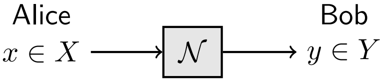
Joint work with
Debbie Leung, Laura Mancinska, William Matthews, Aidan Roy

arXiv:1009.1195

Classical stuff. . .

Classical channels



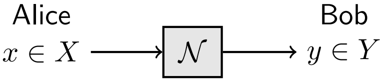
Alice

Bob

$$x \in X \longrightarrow \boxed{\mathcal{N}} \longrightarrow y \in Y$$

QWERTY + “fat fingers”

Classical channels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$x \in X \longrightarrow \boxed{\mathcal{N}} \longrightarrow y \in Y$

Bob


QWERTY + “fat fingers”

Conditional probability distribution

- $\mathcal{N}(x|y) = \Pr($output y | input $x)$ completely characterizes $\mathcal{N}$

Output y

Input x	0	1	2	3	4
0	1/2	1/2			
1		2/3	1/3		
2			1/5	4/5	
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$X = Y = \{0, 1, 2, 3, 4\}$

Classical channels

$$\begin{array}{ccc} \text{Alice} & & \text{Bob} \\ x \in X & \longrightarrow \boxed{\mathcal{N}} \longrightarrow & y \in Y \end{array}$$

QWERTY + “fat fingers”

Conditional probability distribution

- $\mathcal{N}(x|y) = \Pr(\text{output } y \mid \text{input } x)$ completely characterizes $\mathcal{N}$
- $\mathcal{N} \otimes \mathcal{M}$ corresponds to one parallel use of $\mathcal{N}$ and $\mathcal{M}$

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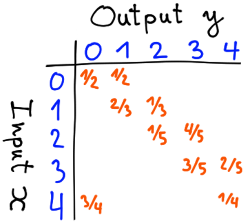
$$\mathcal{N} \otimes \mathcal{M} = \left(\begin{array}{c|c|c} \mathcal{N}_{11}\mathcal{M} & \mathcal{N}_{12}\mathcal{M} & \cdots \\ \hline \mathcal{N}_{21}\mathcal{M} & \mathcal{N}_{22}\mathcal{M} & \cdots \\ \hline \vdots & \vdots & \ddots \end{array}\right)$$

Confusability graph

- Inputs $x, x' \in X$ are confusable if $\exists y \in Y$ such that $\mathcal{N}(y|x) > 0$ and $\mathcal{N}(y|x') > 0$



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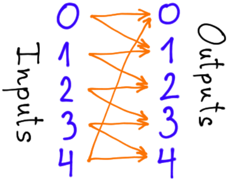

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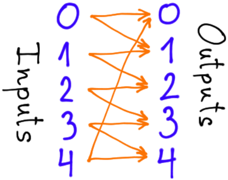

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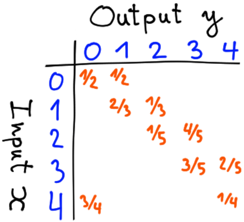

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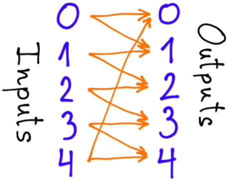

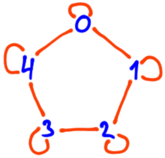

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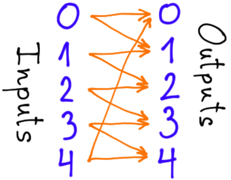

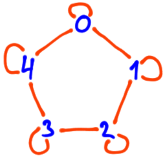

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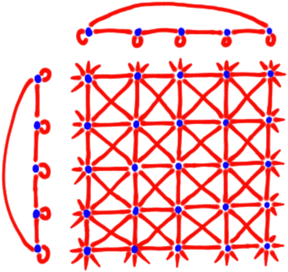

Zero-error capacity

Single-use capacity

- Let $M(\mathcal{N})$ be the number of different messages that can be sent with zero error by a single use of $\mathcal{N}$

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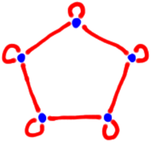

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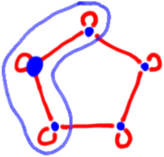

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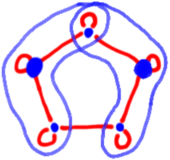

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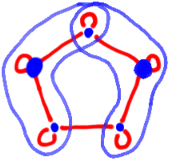

$$M(C_5) = 2$$

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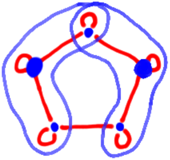
- $M(\mathcal{N}) = \alpha(G_{\mathcal{N}})$, the independence number of $G_{\mathcal{N}}$



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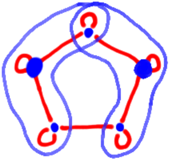

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Asymptotic capacity (Shannon, 1956)

- The zero-error capacity of $\mathcal{N}$ is

$$\Theta(\mathcal{N}) = \lim_{n\to\infty} \sqrt[n]{M(\mathcal{N}^{\otimes n})}$$

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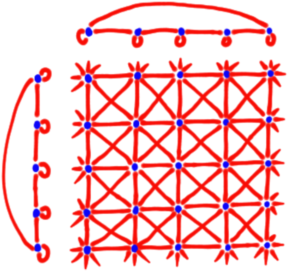

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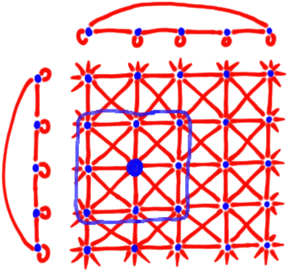

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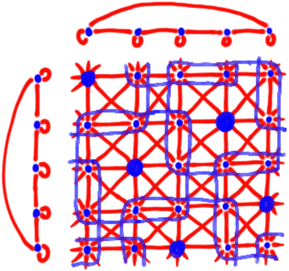

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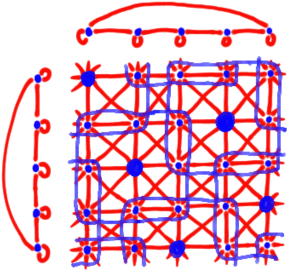

00
12
24
31
43

$$\Theta(C_5) \geq \sqrt{5} \approx 2.236 > M(C_5) = 2$$

An upper bound

Orthogonal representation of a graph

Let $G = (V, E)$ be a graph. Vectors $R = \{r_i \in \mathbb{R}^d : i \in V\}$ form an orthonormal representation of G if

$$r_i^{\mathsf{T}} \cdot r_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } ij \in E \end{cases}$$

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Lovász theta

$$\vartheta(G) = \max_{h,R} \sum_i (h^\mathsf{T} \cdot r_i)^2$$

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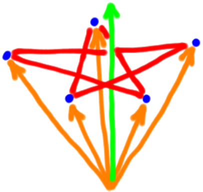
Theorem (Lovász, 1979)

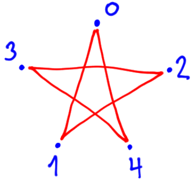
$$\Theta(G) \leq \vartheta(G)$$

Example (pentagon)

An optimal solution for C_5 looks like this:



$$r_k = \begin{pmatrix} \cos\theta \\ \sin\theta\cos\varphi_k \\ \sin\theta\sin\varphi_k \end{pmatrix}$$



where $h = (1, 0, 0)$ and $\cos\theta = \frac{1}{\sqrt[4]{5}}$, $\varphi_k = \frac{2\pi k}{5}$

$$\vartheta(C_5) = \sum_{k=0}^{4} (h^{\mathsf{T}} \cdot r_k)^2 = 5\left(\frac{1}{\sqrt[4]{5}}\right)^2 = \sqrt{5}$$

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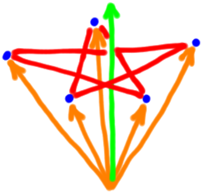

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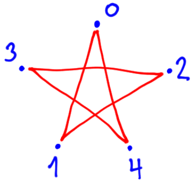

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We conclude that $\Theta(C_5) = \sqrt{5}$

Summary so far. . .

In the context of zero-error communication we don't care about $\mathcal{N}$, but only about the confusability graph $G_{\mathcal{N}}$

Definitions

$$M(G) = \alpha(G)$$

$$\Theta(G) = \lim_{n\to\infty} \sqrt[n]{\alpha(G^{\boxtimes n})}$$

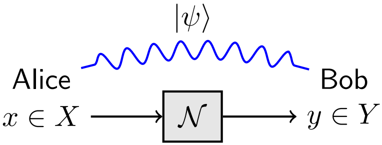
$$\vartheta(G) = \ldots \text{(not so important)} \ldots$$

Relations

$$M(G) \leq \Theta(G) \leq \vartheta(G)$$

Quantum stuff. . .

Classical channels assisted by entanglement



Single-use and asymptotic capacity

- $M_E(\mathcal{N})$ is the number of different messages that can be sent with zero error by a single use of $\mathcal{N}$ and entanglement
- The entanglement-assisted zero-error capacity of $\mathcal{N}$ is

$$\Theta_E(\mathcal{N}) = \lim_{n\to\infty} \sqrt[n]{M_E(\mathcal{N}^{\otimes n})}$$

Entanglement-assisted capacity

Properties

- $M_E(\mathcal{N})$ and $\Theta_E(\mathcal{N})$ are completely determined by $G_{\mathcal{N}}$

---

[1] Beigi [arXiv:1002.2488]

[2] Duan, Severini, Winter [arXiv:1002.2514]

Entanglement-assisted capacity

Properties

- $M_E(\mathcal{N})$ and $\Theta_E(\mathcal{N})$ are completely determined by $G_{\mathcal{N}}$
- $M_E(G) \geq M(G)$ and $\Theta_E(G) \geq \Theta(G)$

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Does there exist a graph G such that $\Theta_E(G) > \Theta(G)$?

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Problem

Does there exist a graph G such that $\Theta_E(G) > \Theta(G)$?

- We need a quantum protocol to lower bound $\Theta_E(G)$
- Lovász bound is not good enough to upper bound $\Theta(G)$

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Lower bound on Θ_E and upper bound on Θ

Theorem (CLMW[3], 2009)

If G has an orthonormal representation in $\mathbb{C}^d$ and its vertices can be partitioned into k disjoint cliques of size d then $\Theta_E(G) = k$

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[3] Cubitt, Leung, Matthews, Winter [arXiv:0911.5300]

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Theorem (Haemers bound, 1979)

$$\Theta(G) \leq R(G) = \min_{B} \operatorname{rank} B$$

where minimization is over all matrices B that fit G

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Lower bound on Θ_E and upper bound on Θ

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If G has an orthonormal representation in $\mathbb{C}^d$ and its vertices can be partitioned into k disjoint cliques of size d then $\Theta_E(G) = k$

Theorem (Haemers bound, 1979)

$$\Theta(G) \leq R(G) = \min_{B} \operatorname{rank} B$$

where minimization is over all matrices B that fit G

Definition

An $|V| \times |V|$ matrix B (over *any* field) fits graph $G = (V, E)$ if $b_{ii} \neq 0$ and $b_{ij} = 0$ if $ij \notin E$

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[3] Cubitt, Leung, Matthews, Winter [arXiv:0911.5300]

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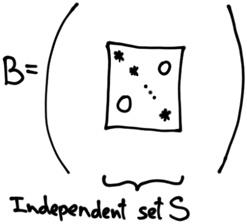
where B fits G, i.e., $b_{ii} \neq 0$ and $b_{ij} = 0$ if $ij \notin E$

Proof

Let S be a maximal independent set in G. If B fits G, then $B_{ij} = 0$ for all $i \neq j \in S$ while the diagonal entries are non-zero. Hence, B has full rank on a subspace of dimension $|S|$ and thus $\operatorname{rank}(B) \geq |S| = \alpha(G)$. As this is true for any B that fits G, we get $R(B) \geq \alpha(G)$.



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If B_1 fits G_1 and B_2 fits G_2 then $B_1 \otimes B_2$ fits $G_1 \boxtimes G_2$, and $\operatorname{rank}(B_1 \otimes B_2) = \operatorname{rank}(B_1) \operatorname{rank}(B_2)$. A non-product matrix can give only a better value so $R(G_1 \boxtimes G_2) \leq R(G_1)R(G_2)$.

Symplectic graphs $\mathrm{sp}(2n, \mathbb{F}_2)$

Definition

Symplectic graph $\mathrm{sp}(2n, \mathbb{F}_2)$ is the orthogonality graph (with respect to the symplectic inner product) of vectors in $\mathbb{F}_2^{2n}$

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($\geq$) by explicitly constructing an independent set of size $2n + 1$

($\leq$) by finding a $(2n + 1)$-dimensional orthonormal representation over $\mathbb{F}_2$ and using Haemers bound

Entanglement-assisted capacity of $\mathrm{sp}(2n, \mathbb{F}_2)$

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The only thing we need now is a low-dimensional orthonormal representation of $\mathrm{sp}(2n, \mathbb{F}_2)$ over $\mathbb{C}$...

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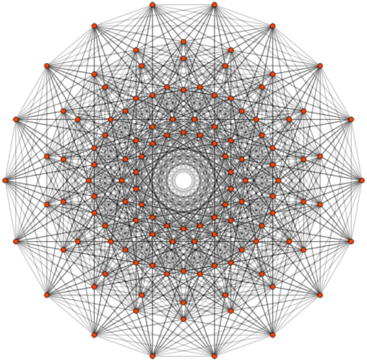
The great coincidence…

It turns out that $\mathrm{sp}(6, \mathbb{F}_2)$ is the orthogonality graph of the root system of the exceptional Lie algebra E_7!

E_7



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- Sometimes you just need luck. . .

Thank you for your attention!